1998 Calculus BC Scoring Guidelines

6. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x–coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with initial condition $x(0) = -4$.

(a) Find $x(t)$ in terms of t.

(b) Find $\frac{dy}{dt}$ in terms of t.

(c) Find the location and speed of the particle at time $t = 4$.

(a) $x(t) = \int \frac{1}{\sqrt{2t+1}}\,dt$

$x(t) = \sqrt{2t+1} + C$

$x(0) = -4 = 1 + C \implies C = -5$

$x(t) = \sqrt{2t+1} - 5$

3 $\begin{cases} 1\text{: } x(t) = \int \frac{dt}{\sqrt{2t+1}} \\ 1\text{: } x(t) = \sqrt{2t+1} + C \\ 1\text{: evaluates } C \end{cases}$

(b) $y = x^3 - 3x$

$\frac{dy}{dt} = 3x^2\frac{dx}{dt} - 3\frac{dx}{dt}$

$= (3x^2 - 3)\frac{dx}{dt}$

$= \left[3\left(\sqrt{2t+1} - 5\right)^2 - 3\right]\left[\frac{1}{\sqrt{2t+1}}\right]$

2: answer

<−1> each error

Note: failure to express $\frac{dy}{dt}$ solely in terms of t is a single error

(c) $x(4) = \sqrt{9} - 5 = -2$

$y(4) = (-2)^3 - 3(-2) = -2$

Location at $t = 4$ is $(-2, -2)$

$\left.\frac{dx}{dt}\right|_{t=4} = \frac{1}{3}$

$\left.\frac{dy}{dt}\right|_{t=4} = \frac{3(3-5)^2 - 3}{3} = 3$

Speed $= \sqrt{\left(\frac{1}{3}\right)^2 + 3^2} = \sqrt{\frac{82}{9}} = 3.018$

4 $\begin{cases} 1\text{: position} \\ 1\text{: evaluates } \frac{dx}{dt} \text{ and } \frac{dy}{dt} \text{ at } t = 4 \\ 1\text{: uses speed formula} \\ 1\text{: answer} \end{cases}$

BC–1 1999

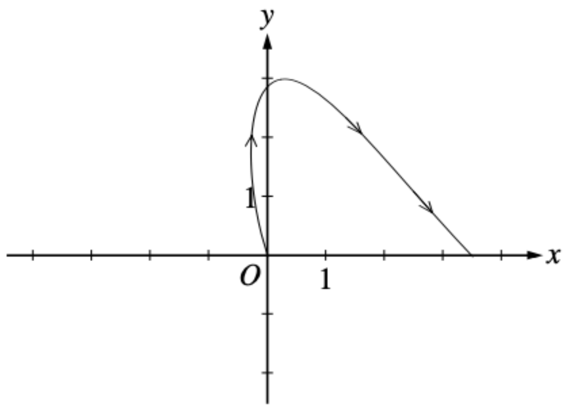
1. A particle moves in the xy–plane so that its position at any time t, $0 \le t \le \pi$, is given by $x(t) = \frac{t^2}{2} - \ln(1+t)$ and $y(t) = 3\sin t$.

(a) Sketch the path of the particle in the xy–plane below. Indicate the direction of motion along the path.

(b) At what time t, $0 \le t \le \pi$, does $x(t)$ attain its minimum value? What is the position $(x(t), y(t))$ of the particle at this time?

(c) At what time t, $0 < t < \pi$, is the particle on the y–axis? Find the speed and the acceleration vector of the particle at this time.

(a)



$2 \begin{cases} 1\text{: graph} \\ 1\text{: direction} \end{cases}$

(b) $x'(t) = t - \frac{1}{1+t} = 0$

$t^2 + t - 1 = 0$

$t = \frac{-1+\sqrt{5}}{2}$ or $t = 0.618$ in $[0, \pi]$

$x(0.618) = -0.290 \qquad y(0.618) = 1.738$

$3 \begin{cases} 1\text{: } x'(t) = 0 \\ 1\text{: solution for } t \\ 1\text{: position} \end{cases}$

(c) $x(t) = \frac{t^2}{2} - \ln(1+t) = 0$

$t = 1.285$ or 1.286

$x'(t) = t - \frac{1}{1+t} \qquad y'(t) = 3\cos t$

speed $= \sqrt{(x'(1.286))^2 + (y'(1.286))^2} = 1.196$

$x''(t) = 1 + \frac{1}{(1+t)^2} \qquad y''(t) = -3\sin t$

acceleration vector $= \langle x''(1.286), y''(1.286) \rangle$
$= \langle 1.191, -2.879 \rangle$

$4 \begin{cases} 1\text{: } x(t) = 0 \\ 1\text{: solution for } t \\ 1\text{: speed} \\ 1\text{: acceleration vector} \end{cases}$

A moving particle has position $(x(t), y(t))$ at time t. The position of the particle at time $t = 1$ is (2,6) and the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.

(a) Find the acceleration vector at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t), y(t))$ have a slope of 8?

(d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

(a) acceleration vector $= (x''(t), y''(t)) = \left(\frac{2}{t^3}, -\frac{2}{t^3}\right)$

$(x''(3), y''(3)) = \left(\frac{2}{27}, -\frac{2}{27}\right)$

$2 \begin{cases} 1: \text{ components of acceleration vector as a function of } t \\ 1: \text{ acceleration vector at } t = 3 \end{cases}$

(b) $(x(t), y(t)) = \left(t + \frac{1}{t} + C_1,\ 2t - \frac{1}{t} + C_2\right)$

$(2, 6) = (x(1), y(1)) = (2 + C_1, 1 + C_2)$

$C_1 = 0,\ C_2 = 5$

$(x(3), y(3)) = \left(3 + \frac{1}{3}, 6 - \frac{1}{3} + 5\right) = \left(\frac{10}{3}, \frac{32}{3}\right)$

$3 \begin{cases} 1: \text{ antidifferentiation} \\ 1: \text{ uses initial condition at } t = 1 \\ 1: \text{ position at } t = 3 \end{cases}$

Note: max 1/3 [1–0–0] if no constants of integration

(c) $\frac{dy}{dx} = \dfrac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 8$

$2 + \frac{1}{t^2} = 8\left(1 - \frac{1}{t^2}\right); \quad t^2 = \frac{9}{6}$

$t = \sqrt{\frac{3}{2}}$

$2 \begin{cases} 1: \frac{dy}{dx} = 8 \text{ as equation in } t \\ 1: \text{ solution for } t \end{cases}$

(d) $\lim\limits_{t \to \infty} \frac{dy}{dx} = \lim\limits_{t \to \infty} \dfrac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 2$

– or –

Since $x(t) \to \infty$ as $t \to \infty$, the slope of the line is

$\lim\limits_{t \to \infty} \frac{y(t)}{x(t)} = \lim\limits_{t \to \infty} \dfrac{2t - \frac{1}{t} + 5}{t + \frac{1}{t}} = 2$

$2 \begin{cases} 1: \text{ considers limit of } \frac{dy}{dx} \text{ or } \frac{y(t)}{x(t)} \\ 1: \text{ answer} \end{cases}$

Note: 0/2 if no consideration of limit

AP® CALCULUS BC
2001 SCORING GUIDELINES

Question 1

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \cos\left(t^3\right) \text{ and } \frac{dy}{dt} = 3\sin\left(t^2\right)$$

for $0 \le t \le 3$. At time $t = 2$, the object is at position (4,5).

(a) Write an equation for the line tangent to the curve at (4,5).

(b) Find the speed of the object at time $t = 2$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

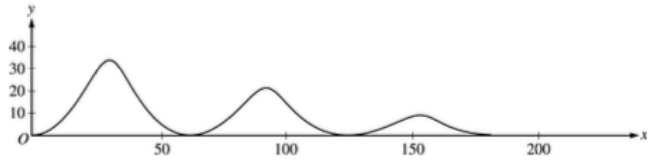
(d) Find the position of the object at time $t = 3$.

(a) $\frac{dy}{dx} = \frac{3\sin\left(t^2\right)}{\cos\left(t^3\right)}$ $\left.\frac{dy}{dx}\right\vert_{t=2} = \frac{3\sin\left(2^2\right)}{\cos\left(2^3\right)} = 15.604$ $y - 5 = 15.604(x - 4)$	1 : tangent line
(b) Speed $= \sqrt{\cos^2(8) + 9\sin^2(4)} = 2.275$	1 : answer
(c) Distance $= \int_0^1 \sqrt{\cos^2\left(t^3\right) + 9\sin^2\left(t^2\right)}\, dt$ $= 1.458$	3 : { 2 : distance integral < −1 > each integrand error < −1 > error in limits 1 : answer }
(d) $x(3) = 4 + \int_2^3 \cos\left(t^3\right) dt = 3.953$ or 3.954 $y(3) = 5 + \int_2^3 3\sin\left(t^2\right) dt = 4.906$	4 : { 1 : definite integral for x 1 : answer for $x(3)$ 1 : definite integral for y 1 : answer for $y(3)$ }

AP® CALCULUS BC 2002 SCORING GUIDELINES

Question 3

The figure above shows the path traveled by a roller coaster car over the time interval $0 \le t \le 18$ seconds. The position of the car at time t seconds can be modeled parametrically by $x(t) = 10t + 4\sin t,\ y(t) = (20 - t)(1 - \cos t)$,



where x and y are measured in meters. The derivatives of these functions are given by

$x'(t) = 10 + 4\cos t,\ y'(t) = (20 - t)\sin t + \cos t - 1.$

(a) Find the slope of the path at time $t = 2$. Show the computations that lead to your answer.

(b) Find the acceleration vector of the car at the time when the car's horizontal position is $x = 140$.

(c) Find the time t at which the car is at its maximum height, and find the speed, in m/sec, of the car at this time.

(d) For $0 < t < 18$, there are two times at which the car is at ground level $(y = 0)$. Find these two times and write an expression that gives the average speed, in m/sec, of the car between these two times. Do not evaluate the expression.

(a) Slope $= \left.\dfrac{dy}{dx}\right|_{t=2} = \dfrac{y'(2)}{x'(2)} = \dfrac{18\sin 2 + \cos 2 - 1}{10 + 4\cos 2}$

$= 1.793$ or 1.794

1 : answer using $\dfrac{dy}{dx} = \dfrac{dy}{dt}\Big/\dfrac{dx}{dt}$

(b) $x(t) = 10t + 4\sin t = 140;\ t_0 = 13.647083$

$x''(t_0) = -3.529,\ y''(t_0) = 1.225$ or 1.226

Acceleration vector is $\langle -3.529, 1.225 \rangle$

or $\langle -3.529, 1.226 \rangle$

2 {
1 : identifies acceleration vector as derivative of velocity vector
1 : computes acceleration vector when $x = 140$

(c) $y'(t) = (20 - t)\sin t + \cos t - 1 = 0$

$t_1 = 3.023$ or 3.024 at maximum height

Speed $= \sqrt{\left(x'(t_1)\right)^2 + \left(y'(t_1)\right)^2} = |x'(t_1)|$

$= 6.027$ or 6.028

3 {
1 : sets $y'(t) = 0$
1 : selects first $t > 0$
1 : speed

(d) $y(t) = 0$ when $t = 2\pi$ and $t = 4\pi$

Average speed $= \dfrac{1}{2\pi}\displaystyle\int_{2\pi}^{4\pi} \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$

$= \dfrac{1}{2\pi}\displaystyle\int_{2\pi}^{4\pi} \sqrt{(10 + 4\cos t)^2 + ((20 - t)\sin t + \cos t - 1)^2}\, dt$

3 {
1 : $t = 2\pi, t = 4\pi$
1 : limits and constant
1 : integrand

AP® CALCULUS BC
2002 SCORING GUIDELINES (Form B)

Question 1

A particle moves in the xy-plane so that its position at any time t, for $-\pi \le t \le \pi$, is given by $x(t) = \sin(3t)$ and $y(t) = 2t$.

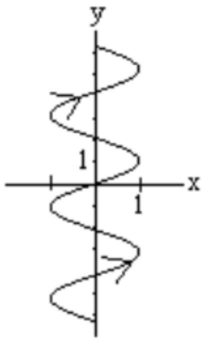
(a) Sketch the path of the particle in the xy-plane provided. Indicate the direction of motion along the path.

(b) Find the range of $x(t)$ and the range of $y(t)$.

(c) Find the smallest positive value of t for which the x-coordinate of the particle is a local maximum. What is the speed of the particle at this time?

(d) Is the distance traveled by the particle from $t = -\pi$ to $t = \pi$ greater than 5π? Justify your answer.

(a)



2 $\begin{cases} 1 : \text{graph} \\ \quad \text{three cycles of sine} \\ \quad x \text{ between } -1 \text{ and } 1 \\ \quad y \text{ between } -2\pi \text{ and } 2\pi \\ 1 : \text{direction} \end{cases}$

(b) $-1 \le x(t) \le 1$

$-2\pi \le y(t) \le 2\pi$

2 $\begin{cases} 1 : \text{closed interval for } x(t) \\ 1 : \text{closed interval for } y(t) \end{cases}$

(c) $x'(t) = 3\cos 3t = 0$

$3t = \dfrac{\pi}{2};\ t = \dfrac{\pi}{6}$

Speed $= \sqrt{9\cos^2(3t) + 4}$

At $t = \dfrac{\pi}{6}$,

Speed $= \sqrt{9\cos^2\left(\dfrac{\pi}{2}\right) + 4} = 2$

3 $\begin{cases} 1 : x'(t) = 3\cos 3t = 0 \\ 1 : \text{solves for } t \\ 1 : \text{speed at student's time} \end{cases}$

(d) Distance $= \displaystyle\int_{-\pi}^{\pi} \sqrt{9\cos^2(3t) + 4}\, dt$

$= 17.973 > 5\pi$

2 $\begin{cases} 1 : \text{integral for distance} \\ 1 : \text{conclusion with justification} \end{cases}$

AP® CALCULUS BC
2003 SCORING GUIDELINES (Form B)

Question 4

A particle moves in the xy-plane so that the position of the particle at any time t is given by

$$x(t) = 2e^{3t} + e^{-7t} \quad \text{and} \quad y(t) = 3e^{3t} - e^{-2t}.$$

(a) Find the velocity vector for the particle in terms of t, and find the speed of the particle at time $t = 0$.

(b) Find $\frac{dy}{dx}$ in terms of t, and find $\lim_{t\to\infty} \frac{dy}{dx}$.

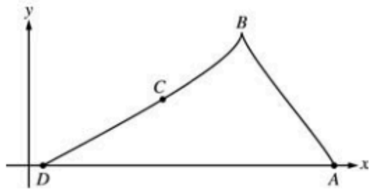
(c) Find each value t at which the line tangent to the path of the particle is horizontal, or explain why none exists.

(d) Find each value t at which the line tangent to the path of the particle is vertical, or explain why none exists.

Solution	Points
(a) $x'(t) = 6e^{3t} - 7e^{-7t}$ $y'(t) = 9e^{3t} + 2e^{-2t}$ Velocity vector is $< 6e^{3t} - 7e^{-7t},\ 9e^{3t} + 2e^{-2t} >$ Speed $= \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{(-1)^2 + 11^2}$ $= \sqrt{122}$	$3: \begin{cases} 1: x'(t) \\ 1: y'(t) \\ 1: \text{speed} \end{cases}$
(b) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}}$ $\lim_{t\to\infty} \frac{dy}{dx} = \lim_{t\to\infty} \frac{9e^{3t} + 2e^{-2t}}{6e^{3t} - 7e^{-7t}} = \frac{9}{6} = \frac{3}{2}$	$2: \begin{cases} 1: \frac{dy}{dx} \text{ in terms of } t \\ 1: \text{limit} \end{cases}$
(c) Need $y'(t) = 0$, but $9e^{3t} + 2e^{-2t} > 0$ for all t, so none exists.	$2: \begin{cases} 1: \text{considers } y'(t) = 0 \\ 1: \text{explains why none exists} \end{cases}$
(d) Need $x'(t) = 0$ and $y'(t) \neq 0$. $6e^{3t} = 7e^{-7t}$ $e^{10t} = \frac{7}{6}$ $t = \frac{1}{10}\ln\left(\frac{7}{6}\right)$	$2: \begin{cases} 1: \text{considers } x'(t) = 0 \\ 1: \text{solution} \end{cases}$

AP® CALCULUS BC
2003 SCORING GUIDELINES

Question 2



A particle starts at point A on the positive x-axis at time $t = 0$ and travels along the curve from A to B to C to D, as shown above. The coordinates of the particle's position $\left(x(t), y(t)\right)$ are differentiable functions of t, where

$x'(t) = \dfrac{dx}{dt} = -9\cos\left(\dfrac{\pi t}{6}\right)\sin\left(\dfrac{\pi\sqrt{t+1}}{2}\right)$ and $y'(t) = \dfrac{dy}{dt}$ is not explicitly given.

At time $t = 9$, the particle reaches its final position at point D on the positive x-axis.

(a) At point C, is $\dfrac{dy}{dt}$ positive? At point C, is $\dfrac{dx}{dt}$ positive? Give a reason for each answer.

(b) The slope of the curve is undefined at point B. At what time t is the particle at point B?

(c) The line tangent to the curve at the point $\left(x(8), y(8)\right)$ has equation $y = \dfrac{5}{9}x - 2$. Find the velocity vector and the speed of the particle at this point.

(d) How far apart are points A and D, the initial and final positions, respectively, of the particle?

(a) At point C, $\dfrac{dy}{dt}$ is not positive because $y(t)$ is decreasing along the arc BD as t increases.

At point C, $\dfrac{dx}{dt}$ is not positive because $x(t)$ is decreasing along the arc BD as t increases.

$2: \begin{cases} 1: \dfrac{dy}{dt} \text{ not positive with reason} \\ 1: \dfrac{dx}{dt} \text{ not positive with reason} \end{cases}$

(b) $\dfrac{dx}{dt} = 0$; $\cos\left(\dfrac{\pi t}{6}\right) = 0$ or $\sin\left(\dfrac{\pi\sqrt{t+1}}{2}\right) = 0$

$\dfrac{\pi t}{6} = \dfrac{\pi}{2}$ or $\dfrac{\pi\sqrt{t+1}}{2} = \pi$; $t = 3$ for both.

Particle is at point B at $t = 3$.

$2: \begin{cases} 1: \text{sets } \dfrac{dx}{dt} = 0 \\ 1: t = 3 \end{cases}$

(c) $x'(8) = -9\cos\left(\dfrac{4\pi}{3}\right)\sin\left(\dfrac{3\pi}{2}\right) = -\dfrac{9}{2}$

$\dfrac{y'(8)}{x'(8)} = \dfrac{dy}{dx} = \dfrac{5}{9}$

$y'(8) = \dfrac{5}{9}x'(8) = -\dfrac{5}{2}$

The velocity vector is $< -4.5, -2.5 >$.

Speed $= \sqrt{4.5^2 + 2.5^2} = 5.147$ or 5.148

$3: \begin{cases} 1: x'(8) \\ 1: y'(8) \\ 1: \text{speed} \end{cases}$

(d) $x(9) - x(0) = \displaystyle\int_0^9 x'(t)\,dt$

$= -39.255$

The initial and final positions are 39.255 apart.

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

AP® CALCULUS BC
2004 SCORING GUIDELINES (Form B)

Question 1

A particle moving along a curve in the plane has position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sqrt{t^4 + 9} \text{ and } \frac{dy}{dt} = 2e^t + 5e^{-t}$$

for all real values of t. At time $t = 0$, the particle is at the point $(4, 1)$.

(a) Find the speed of the particle and its acceleration vector at time $t = 0$.
(b) Find an equation of the line tangent to the path of the particle at time $t = 0$.
(c) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.
(d) Find the x-coordinate of the position of the particle at time $t = 3$.

(a) At time $t = 0$:

Speed $= \sqrt{x'(0)^2 + y'(0)^2} = \sqrt{3^2 + 7^2} = \sqrt{58}$

Acceleration vector $= \langle x''(0), y''(0)\rangle = \langle 0, -3\rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration vector} \end{cases}$

(b) $\dfrac{dy}{dx} = \dfrac{y'(0)}{x'(0)} = \dfrac{7}{3}$

Tangent line is $y = \frac{7}{3}(x - 4) + 1$

$2 : \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line} \end{cases}$

(c) Distance $= \displaystyle\int_0^3 \sqrt{\left(\sqrt{t^4 + 9}\right)^2 + \left(2e^t + 5e^{-t}\right)^2}\, dt$

$= 45.226$ or 45.227

$3 : \begin{cases} 2 : \text{distance integral} \\ \quad \langle -1 \rangle \text{ each integrand error} \\ \quad \langle -1 \rangle \text{ error in limits} \\ 1 : \text{answer} \end{cases}$

(d) $x(3) = 4 + \displaystyle\int_0^3 \sqrt{t^4 + 9}\, dt$

$= 17.930$ or 17.931

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

AP® CALCULUS BC
2004 SCORING GUIDELINES

Question 3

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \geq 0$ with $\frac{dx}{dt} = 3 + \cos(t^2)$. The derivative $\frac{dy}{dt}$ is not explicitly given. At time $t = 2$, the object is at position $(1, 8)$.

(a) Find the x-coordinate of the position of the object at time $t = 4$.

(b) At time $t = 2$, the value of $\frac{dy}{dt}$ is -7. Write an equation for the line tangent to the curve at the point $(x(2), y(2))$.

(c) Find the speed of the object at time $t = 2$.

(d) For $t \geq 3$, the line tangent to the curve at $(x(t), y(t))$ has a slope of $2t + 1$. Find the acceleration vector of the object at time $t = 4$.

(a)
$$x(4) = x(2) + \int_2^4 \left(3 + \cos(t^2)\right) dt$$
$$= 1 + \int_2^4 \left(3 + \cos(t^2)\right) dt = 7.132 \text{ or } 7.133$$

$3 : \begin{cases} 1 : \int_2^4 \left(3 + \cos(t^2)\right) dt \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{cases}$

(b)
$$\left.\frac{dy}{dx}\right|_{t=2} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=2} = \frac{-7}{3 + \cos 4} = -2.983$$
$$y - 8 = -2.983(x - 1)$$

$2 : \begin{cases} 1 : \text{finds } \left.\frac{dy}{dx}\right|_{t=2} \\ 1 : \text{equation} \end{cases}$

(c) The speed of the object at time $t = 2$ is
$$\sqrt{(x'(2))^2 + (y'(2))^2} = 7.382 \text{ or } 7.383.$$

$1 : \text{answer}$

(d)
$$x''(4) = 2.303$$
$$y'(t) = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = (2t + 1)\left(3 + \cos(t^2)\right)$$
$$y''(4) = 24.813 \text{ or } 24.814$$
The acceleration vector at $t = 4$ is $\langle 2.303, 24.813 \rangle$ or $\langle 2.303, 24.814 \rangle$.

$3 : \begin{cases} 1 : x''(4) \\ 1 : \frac{dy}{dt} \\ 1 : \text{answer} \end{cases}$

AP® CALCULUS BC
2005 SCORING GUIDELINES (Form B)

Question 1

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \ge 0$ with

$$\frac{dx}{dt} = 12t - 3t^2 \text{ and } \frac{dy}{dt} = \ln\left(1 + (t-4)^4\right).$$

At time $t = 0$, the object is at position $(-13, 5)$. At time $t = 2$, the object is at point P with x-coordinate 3.

(a) Find the acceleration vector at time $t = 2$ and the speed at time $t = 2$.

(b) Find the y-coordinate of P.

(c) Write an equation for the line tangent to the curve at P.

(d) For what value of t, if any, is the object at rest? Explain your reasoning.

(a) $x''(2) = 0,\ y''(2) = -\dfrac{32}{17} = -1.882$

$a(2) = \langle 0, -1.882 \rangle$

Speed $= \sqrt{12^2 + (\ln(17))^2} = 12.329$ or 12.330

$2: \begin{cases} 1: \text{acceleration vector} \\ 1: \text{speed} \end{cases}$

(b) $y(t) = y(0) + \int_0^t \ln\left(1 + (u-4)^4\right) du$

$y(2) = 5 + \int_0^2 \ln\left(1 + (u-4)^4\right) du = 13.671$

$3: \begin{cases} 1: \int_0^2 \ln\left(1 + (u-4)^4\right) du \\ 1: \text{handles initial condition} \\ 1: \text{answer} \end{cases}$

(c) At $t = 2$, slope $= \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{\ln(17)}{12} = 0.236$

$y - 13.671 = 0.236(x - 3)$

$2: \begin{cases} 1: \text{slope} \\ 1: \text{equation} \end{cases}$

(d) $x'(t) = 0$ if $t = 0, 4$

$y'(t) = 0$ if $t = 4$

$t = 4$

$2: \begin{cases} 1: \text{reason} \\ 1: \text{answer} \end{cases}$

AP® CALCULUS BC
2006 SCORING GUIDELINES (Form B)

Question 2

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \tan\left(e^{-t}\right) \text{ and } \frac{dy}{dt} = \sec\left(e^{-t}\right)$$

for $t \ge 0$. At time $t = 1$, the object is at position $(2, -3)$.

(a) Write an equation for the line tangent to the curve at position $(2, -3)$.

(b) Find the acceleration vector and the speed of the object at time $t = 1$.

(c) Find the total distance traveled by the object over the time interval $1 \le t \le 2$.

(d) Is there a time $t \ge 0$ at which the object is on the y-axis? Explain why or why not.

(a) $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec\left(e^{-t}\right)}{\tan\left(e^{-t}\right)} = \frac{1}{\sin\left(e^{-t}\right)}$$

$$\left.\frac{dy}{dx}\right|_{(2,-3)} = \frac{1}{\sin\left(e^{-1}\right)} = 2.780 \text{ or } 2.781$$

$$y + 3 = \frac{1}{\sin\left(e^{-1}\right)}(x - 2)$$

$2 : \begin{cases} 1 : \left.\frac{dy}{dx}\right|_{(2,-3)} \\ 1 : \text{equation of tangent line} \end{cases}$

(b) $x''(1) = -0.42253,\ y''(1) = -0.15196$

$a(1) = \langle -0.423, -0.152 \rangle$ or $\langle -0.422, -0.151 \rangle$.

$$\text{speed} = \sqrt{\left(\sec\left(e^{-1}\right)\right)^2 + \left(\tan\left(e^{-1}\right)\right)^2} = 1.138 \text{ or } 1.139$$

$2 : \begin{cases} 1 : \text{acceleration vector} \\ 1 : \text{speed} \end{cases}$

(c) $$\int_1^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = 1.059$$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $$x(0) = x(1) - \int_0^1 x'(t)\, dt = 2 - 0.775553 > 0$$

The particle starts to the right of the y-axis.
Since $x'(t) > 0$ for all $t \ge 0$, the object is always moving to the right and thus is never on the y-axis.

$3 : \begin{cases} 1 : x(0) \text{ expression} \\ 1 : x'(t) > 0 \\ 1 : \text{conclusion and reason} \end{cases}$

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2006 SCORING GUIDELINES

Question 3

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t, where

$$\frac{dx}{dt} = \sin^{-1}\left(1 - 2e^{-t}\right) \text{ and } \frac{dy}{dt} = \frac{4t}{1+t^3}$$

for $t \geq 0$. At time $t = 2$, the object is at the point $(6, -3)$. (Note: $\sin^{-1} x = \arcsin x$)

(a) Find the acceleration vector and the speed of the object at time $t = 2$.

(b) The curve has a vertical tangent line at one point. At what time t is the object at this point?

(c) Let $m(t)$ denote the slope of the line tangent to the curve at the point $(x(t), y(t))$. Write an expression for $m(t)$ in terms of t and use it to evaluate $\lim_{t\to\infty} m(t)$.

(d) The graph of the curve has a horizontal asymptote $y = c$. Write, but do not evaluate, an expression involving an improper integral that represents this value c.

(a) $a(2) = \langle 0.395 \text{ or } 0.396, -0.741 \text{ or } -0.740\rangle$ Speed $= \sqrt{x'(2)^2 + y'(2)^2} = 1.207$ or 1.208	2 : { 1 : acceleration 1 : speed
(b) $\sin^{-1}\left(1 - 2e^{-t}\right) = 0$ $1 - 2e^{-t} = 0$ $t = \ln 2 = 0.693$ and $\frac{dy}{dt} \neq 0$ when $t = \ln 2$	2 : { 1 : $x'(t) = 0$ 1 : answer
(c) $m(t) = \frac{4t}{1+t^3} \cdot \frac{1}{\sin^{-1}\left(1-2e^{-t}\right)}$ $\lim_{t\to\infty} m(t) = \lim_{t\to\infty}\left(\frac{4t}{1+t^3} \cdot \frac{1}{\sin^{-1}\left(1-2e^{-t}\right)}\right)$ $= 0\left(\frac{1}{\sin^{-1}(1)}\right) = 0$	2 : { 1 : $m(t)$ 1 : limit value
(d) Since $\lim_{t\to\infty} x(t) = \infty$, $c = \lim_{t\to\infty} y(t) = -3 + \int_2^\infty \frac{4t}{1+t^3}\,dt$	3 : { 1: integrand 1: limits 1: initial value consistent with lower limit

AP® CALCULUS BC
2007 SCORING GUIDELINES (Form B)

Question 2

An object moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \arctan\left(\frac{t}{1+t}\right) \text{ and } \frac{dy}{dt} = \ln\left(t^2+1\right)$$

for $t \geq 0$. At time $t = 0$, the object is at position $(-3, -4)$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the speed of the object at time $t = 4$.

(b) Find the total distance traveled by the object over the time interval $0 \leq t \leq 4$.

(c) Find $x(4)$.

(d) For $t > 0$, there is a point on the curve where the line tangent to the curve has slope 2. At what time t is the object at this point? Find the acceleration vector at this point.

(a) Speed $= \sqrt{x'(4)^2 + y'(4)^2} = 2.912$

1 : speed at $t = 4$

(b) Distance $= \displaystyle\int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 6.423$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $x(4) = x(0) + \displaystyle\int_0^4 x'(t)\,dt$

$= -3 + 2.10794 = -0.892$

$3 : \begin{cases} 2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } x(0) = -3 \end{cases} \\ 1 : \text{answer} \end{cases}$

(d) The slope is 2, so $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2$, or $\ln\left(t^2+1\right) = 2\arctan\left(\frac{t}{1+t}\right)$.

Since $t > 0$, $t = 1.35766$. At this time, the acceleration is $\langle x''(t), y''(t)\rangle\big|_{t=1.35766} = \langle 0.135, 0.955\rangle$.

$3 : \begin{cases} 1 : \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2 \\ 1 : t\text{-value} \\ 1 : \text{values for } x'' \text{ and } y'' \end{cases}$

AP® CALCULUS BC
2008 SCORING GUIDELINES (Form B)

Question 1

A particle moving along a curve in the xy-plane has position $(x(t),\ y(t))$ at time $t \geq 0$ with

$$\frac{dx}{dt} = \sqrt{3t} \text{ and } \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right).$$

The particle is at position $(1, 5)$ at time $t = 4$.

(a) Find the acceleration vector at time $t = 4$.

(b) Find the y-coordinate of the position of the particle at time $t = 0$.

(c) On the interval $0 \leq t \leq 4$, at what time does the speed of the particle first reach 3.5 ?

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 4$.

(a) $a(4) = \langle x''(4),\ y''(4)\rangle = \langle 0.433, -11.872\rangle$

$1:$ answer

(b) $y(0) = 5 + \int_4^0 3\cos\left(\frac{t^2}{2}\right) dt = 1.600$ or 1.601

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } y(4) = 5 \\ 1 : \text{answer} \end{cases}$$

(c) Speed $= \sqrt{(x'(t))^2 + (y'(t))^2}$

$$= \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} = 3.5$$

The particle first reaches this speed when $t = 2.225$ or 2.226.

$$3 : \begin{cases} 1 : \text{expression for speed} \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$$

(d) $\int_0^4 \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)}\, dt = 13.182$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

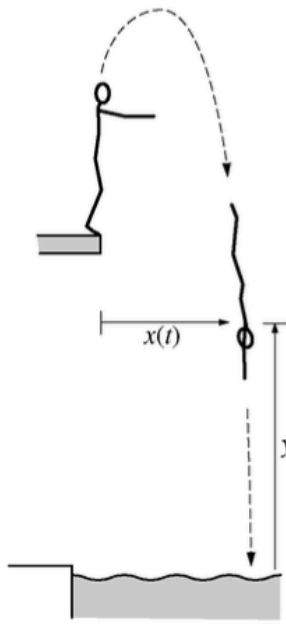
AP® CALCULUS BC
2009 SCORING GUIDELINES

Question 3

A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time t seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \text{ and } \frac{dy}{dt} = 3.6 - 9.8t,$$

for $0 \le t \le A$, where A is the time that the diver's shoulders enter the water.



Note: Figure not drawn to scale.

(a) Find the maximum vertical distance from the water surface to the diver's shoulders.

(b) Find A, the time that the diver's shoulders enter the water.

(c) Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.

(d) Find the angle θ, $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.

(a) $\frac{dy}{dt} = 0$ only when $t = 0.36735$. Let $b = 0.36735$. The maximum vertical distance from the water surface to the diver's shoulders is $y(b) = 11.4 + \int_0^b \frac{dy}{dt}\,dt = 12.061$ meters. Alternatively, $y(t) = 11.4 + 3.6t - 4.9t^2$, so $y(b) = 12.061$ meters.	$3: \begin{cases} 1: \text{considers } \frac{dy}{dt} = 0 \\ 1: \text{integral or } y(t) \\ 1: \text{answer} \end{cases}$
(b) $y(A) = 11.4 + \int_0^A \frac{dy}{dt}\,dt = 11.4 + 3.6A - 4.9A^2 = 0$ when $A = 1.936$ seconds.	$2: \begin{cases} 1: \text{equation} \\ 1: \text{answer} \end{cases}$
(c) $\int_0^A \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 12.946$ meters	$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$
(d) At time A, $\frac{dy}{dx} = \left.\frac{dy/dt}{dx/dt}\right\rvert_{t=A} = -19.21913$. The angle between the path of the diver and the water is $\tan^{-1}(19.21913) = 1.518$ or 1.519.	$2: \begin{cases} 1: \frac{dy}{dx} \text{ at time } A \\ 1: \text{answer} \end{cases}$

AP® CALCULUS BC
2010 SCORING GUIDELINES (Form B)

Question 2

The velocity vector of a particle moving in the plane has components given by

$$\frac{dx}{dt} = 14\cos\left(t^2\right)\sin\left(e^t\right) \text{ and } \frac{dy}{dt} = 1 + 2\sin\left(t^2\right), \text{ for } 0 \le t \le 1.5.$$

At time $t = 0$, the position of the particle is $(-2, 3)$.

(a) For $0 < t < 1.5$, find all values of t at which the line tangent to the path of the particle is vertical.

(b) Write an equation for the line tangent to the path of the particle at $t = 1$.

(c) Find the speed of the particle at $t = 1$.

(d) Find the acceleration vector of the particle at $t = 1$.

(a) The tangent line is vertical when $x'(t) = 0$ and $y'(t) \ne 0$. On $0 < t < 1.5$, this happens at $t = 1.253$ and $t = 1.144$ or 1.145.	$2: \begin{cases} 1: \text{sets } \frac{dx}{dy} = 0 \\ 1: \text{answer} \end{cases}$
(b) $\left.\frac{dy}{dx}\right\rvert_{t=1} = \frac{y'(1)}{x'(1)} = 0.863447$ $x(1) = -2 + \int_0^1 x'(t)\,dt = 9.314695$ $y(1) = 3 + \int_0^1 y'(t)\,dt = 4.620537$ The line tangent to the path of the particle at $t = 1$ has equation $y = 4.621 + 0.863(x - 9.315)$.	$4: \begin{cases} 1: \left.\frac{dy}{dx}\right\rvert_{t=1} \\ 1: x(1) \\ 1: y(1) \\ 1: \text{equation} \end{cases}$
(c) Speed $= \sqrt{(x'(1))^2 + (y'(1))^2} = 4.105$	1 : answer
(d) Acceleration vector: $\langle x''(1), y''(1)\rangle = \langle -28.425, 2.161\rangle$	$2: \begin{cases} 1: x''(1) \\ 1: y''(1) \end{cases}$

AP® CALCULUS BC
2010 SCORING GUIDELINES

Question 3

A particle is moving along a curve so that its position at time t is $(x(t), y(t))$, where $x(t) = t^2 - 4t + 8$ and $y(t)$ is not explicitly given. Both x and y are measured in meters, and t is measured in seconds. It is known that $\frac{dy}{dt} = te^{t-3} - 1$.

(a) Find the speed of the particle at time $t = 3$ seconds.

(b) Find the total distance traveled by the particle for $0 \le t \le 4$ seconds.

(c) Find the time t, $0 \le t \le 4$, when the line tangent to the path of the particle is horizontal. Is the direction of motion of the particle toward the left or toward the right at that time? Give a reason for your answer.

(d) There is a point with x-coordinate 5 through which the particle passes twice. Find each of the following.

(i) The two values of t when that occurs

(ii) The slopes of the lines tangent to the particle's path at that point

(iii) The y-coordinate of that point, given $y(2) = 3 + \frac{1}{e}$

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 2.828$ meters per second	1 : answer
(b) $x'(t) = 2t - 4$ Distance $= \int_0^4 \sqrt{(2t-4)^2 + \left(te^{t-3} - 1\right)^2}\, dt = 11.587$ or 11.588 meters	2 : { 1 : integral 1 : answer
(c) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = 0$ when $te^{t-3} - 1 = 0$ and $2t - 4 \neq 0$ This occurs at $t = 2.20794$. Since $x'(2.20794) > 0$, the particle is moving toward the right at time $t = 2.207$ or 2.208.	3 : { 1 : considers $\frac{dy}{dx} = 0$ 1 : $t = 2.207$ or 2.208 1 : direction of motion with reason
(d) $x(t) = 5$ at $t = 1$ and $t = 3$ At time $t = 1$, the slope is $\left.\frac{dy}{dx}\right\vert_{t=1} = \left.\frac{dy/dt}{dx/dt}\right\vert_{t=1} = 0.432$. At time $t = 3$, the slope is $\left.\frac{dy}{dx}\right\vert_{t=3} = \left.\frac{dy/dt}{dx/dt}\right\vert_{t=3} = 1$. $y(1) = y(3) = 3 + \frac{1}{e} + \int_2^3 \frac{dy}{dt}\, dt = 4$	3 : { 1 : $t = 1$ and $t = 3$ 1 : slopes 1 : y-coordinate

AP® CALCULUS BC
2011 SCORING GUIDELINES

Question 1

At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \geq 0$, $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin(t^2)$. At time $t = 0$, $x(0) = 0$ and $y(0) = -4$.

(a) Find the speed of the particle at time $t = 3$, and find the acceleration vector of the particle at time $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at time $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 13.006$ or 13.007 Acceleration $= \langle x''(3), y''(3) \rangle$ $= \langle 4, -5.466 \rangle$ or $\langle 4, -5.467 \rangle$	2 : { 1 : speed 1 : acceleration
(b) Slope $= \frac{y'(3)}{x'(3)} = 0.031$ or 0.032	1 : answer
(c) $x(3) = 0 + \int_0^3 \frac{dx}{dt}\,dt = 21$ $y(3) = -4 + \int_0^3 \frac{dy}{dt}\,dt = -3.226$ At time $t = 3$, the particle is at position $(21, -3.226)$.	4 : { 2 : x-coordinate 1 : integral 1 : answer 2 : y-coordinate 1 : integral 1 : answer
(d) Distance $= \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 21.091$	2 : { 1 : integral 1 : answer

AP® CALCULUS BC
2012 SCORING GUIDELINES

Question 2

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\frac{dx}{dt} = \frac{\sqrt{t+2}}{e^t}$ and $\frac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

(a) $\left.\frac{dx}{dt}\right|_{t=2} = \frac{2}{e^2}$

Because $\left.\frac{dx}{dt}\right|_{t=2} > 0$, the particle is moving to the right at time $t = 2$.

$\left.\frac{dy}{dx}\right|_{t=2} = \frac{dy/dt|_{t=2}}{dx/dt|_{t=2}} = 3.055$ (or 3.054)

$3 : \begin{cases} 1 : \text{moving to the right with reason} \\ 1 : \text{considers } \frac{dy/dt}{dx/dt} \\ 1 : \text{slope at } t = 2 \end{cases}$

(b) $x(4) = 1 + \int_2^4 \frac{\sqrt{t+2}}{e^t}\,dt = 1.253$ (or 1.252)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = 0.575$ (or 0.574)

Acceleration $= \langle x''(4), y''(4) \rangle$
$= \langle -0.041, 0.989 \rangle$

$2 : \begin{cases} 1 : \text{speed} \\ 1 : \text{acceleration} \end{cases}$

(d) Distance $= \int_2^4 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$
$= 0.651$ (or 0.650)

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

AP® CALCULUS BC
2015 SCORING GUIDELINES

Question 2

At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos\left(t^2\right), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

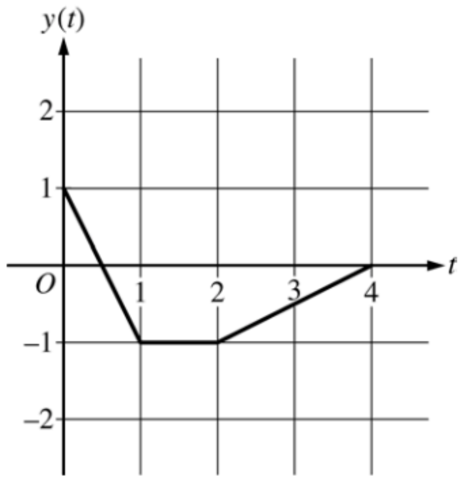
(c) Find the time at which the speed of the particle is 3.

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

(a) $x(2) = 3 + \int_1^2 \cos\left(t^2\right)\, dt = 2.557$ (or 2.556)	$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$
(b) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^{0.5t}}{\cos\left(t^2\right)}$ $\frac{e^{0.5t}}{\cos\left(t^2\right)} = 2$ $t = 0.840$	$2 : \begin{cases} 1 : \text{slope in terms of } t \\ 1 : \text{answer} \end{cases}$
(c) Speed $= \sqrt{\cos^2\left(t^2\right) + e^t}$ $\sqrt{\cos^2\left(t^2\right) + e^t} = 3$ $t = 2.196$ (or 2.195)	$2 : \begin{cases} 1 : \text{speed in terms of } t \\ 1 : \text{answer} \end{cases}$
(d) Distance $= \int_0^1 \sqrt{\cos^2\left(t^2\right) + e^t}\, dt = 1.595$ (or 1.594)	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

AP® CALCULUS BC
2016 SCORING GUIDELINES

Question 2



At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\frac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above. At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

(c) Find the speed of the particle at $t = 3$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

(a) $x(3) = x(0) + \int_0^3 x'(t)\,dt = 5 + 9.377035 = 14.377$

$y(3) = -\frac{1}{2}$

The position of the particle at $t = 3$ is $(14.377, -0.5)$.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

(b) Slope $= \frac{y'(3)}{x'(3)} = \frac{0.5}{9.956376} = 0.05$

1 : slope

(c) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 9.969$ (or 9.968)

$2 : \begin{cases} 1 : \text{expression for speed} \\ 1 : \text{answer} \end{cases}$

(d) Distance $= \int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$

$= \int_0^1 \sqrt{(x'(t))^2 + (-2)^2}\,dt + \int_1^2 \sqrt{(x'(t))^2 + 0^2}\,dt$

$= 2.237871 + 2.112003 = 4.350$ (or 4.349)

$3 : \begin{cases} 1 : \text{expression for distance} \\ 1 : \text{integrals} \\ 1 : \text{answer} \end{cases}$

AP® CALCULUS BC
2018 SCORING GUIDELINES

Question 2

(a) $p'(25) = -1.179$ At a depth of 25 meters, the density of plankton cells is changing at a rate of -1.179 million cells per cubic meter per meter.	$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$
(b) $\int_0^{30} 3p(h)\,dh = 1675.414936$ There are 1675 million plankton cells in the column of water between $h = 0$ and $h = 30$ meters.	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\int_{30}^{K} 3f(h)\,dh$ represents the number of plankton cells, in millions, in the column of water from a depth of 30 meters to a depth of K meters. The number of plankton cells, in millions, in the entire column of water is given by $\int_0^{30} 3p(h)\,dh + \int_{30}^{K} 3f(h)\,dh.$ Because $0 \le f(h) \le u(h)$ for all $h \ge 30$, $3\int_{30}^{K} f(h)\,dh \le 3\int_{30}^{K} u(h)\,dh \le 3\int_{30}^{\infty} u(h)\,dh = 3 \cdot 105 = 315.$ The total number of plankton cells in the column of water is bounded by $1675.415 + 315 = 1990.415 \le 2000$ million.	$3 : \begin{cases} 1 : \text{integral expression} \\ 1 : \text{compares improper integral} \\ 1 : \text{explanation} \end{cases}$
(d) $\int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt = 757.455862$ The total distance traveled by the boat over the time interval $0 \le t \le 1$ is 757.456 (or 757.455) meters.	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{total distance} \end{cases}$